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Experimental and Numerical Investigations on Fracture Process Zone of Rock-Concrete Interface

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ABSTRACT

A crack propagation criterion for a rock-concrete interface is employed to investigate the evolution of the fracture process zone (FPZ) in rock-concrete composite beams under three-point bending (TPB). According to the criterion, cracking initiates along the interface when the difference between the mode I stress intensity factor (SIF) at the crack tip caused by external loading and the one caused by the cohesive stress acting on the fictitious crack surfaces reaches the initial fracture toughness of a rock-concrete interface. From the experimental results of the composite beams with various initial crack lengths but equal depths under TPB, the interface fracture parameters are determined. In addition, the FPZ evolution in a TPB specimen is investigated by using a digital image correlation (DIC) technique. Thus, the fracture processes of the rock-concrete composite beams can be simulated by introducing the initial fracture criterion to determine the crack propagation. By comparing the load versus crack mouth opening displacement (CMOD) curves and FPZ evolution, the numerical and experimental results show a reasonable agreement, which verifies the numerical method developed in this study for analysing the crack propagation along the rock-concrete interface. Finally, based on the numerical results, the effect of ligament length on the FPZ evolution and the variations of the fracture model during crack propagation are discussed for the rock-concrete interface fracture under TPB. The results indicate that ligament length significantly affects the FPZ evolution at the rock-concrete interface under TPB, and the stress intensity factor ratio of mode II to I is influenced by the specimen size during the propagation of the interfacial crack.

Keywords: Rock-concrete interface; Interfacial fracture; FPZ evolution; Crack

45 **propagation; Numerical simulation.**

46

47 ## Nomenclature

a	=	crack length including cohesive crack length
A	=	interfacial area
A_{lig}	=	ligament area of the composite specimen
$CMOD$	=	crack mouth opening displacement
D	=	depth of specimen
DIC	=	digital image correlation
E_i	=	Young's modulus
f_t	=	uniaxial tensile strength
f_c	=	uniaxial compressive strength
FPZ	=	fracture process zone
G_f	=	fracture energy of concrete
$GMTS$	=	Generalised maximum tangential stress criterion
K_I^P	=	stress intensity factor of mode I caused by external loading
K_i^σ	=	of mode I caused by cohesive stress
K_{IC}^{ini}	=	initial stress intensity factor fracture toughness
K_{IRC}^{ini}	=	initial fracture toughness of rock-concrete interface
K_i	=	stress intensity factors
L_{FPZ}^{max}	=	full FPZ length
l_{FPZ}	=	FPZ length
$LEFM$	=	linear elastic fracture mechanics
P_{ini}	=	initial cracking load
P_{max}	=	peak load
P_w	=	self-weight of the composite specimen

TPB = three-point bending

W_0 = area under the load-deformation curve

w = crack opening displacement

w_0 = stress-free crack opening displacement

w_s = crack opening displacement at the breaking point

$\triangle a$ = crack propagation length

δ_x = relative crack surface displacement in x direction

δ_y = relative crack surface displacement in y direction

δ_0 = deformation when load decreases to 0

σ = cohesive stress

σ_s = cohesive stress at the breaking point

INTRODUCTION

The bonded interface between dissimilar materials, such as the interface between a concrete gravity dam and the bedrock, is always a weak link, promoting crack initiation and leading to fracture even under service loads. The linear-elastic fracture mechanics (LEFM) developed by Rice[1] has been extensively accepted as a suitable technique to analyse and evaluate the potential fracture at a bi-material interface. Based on LEFM, some interfacial fracture parameters such as fracture energy and fracture toughness were investigated through experimental and numerical studies[2-4]. The experimental studies indicated that the magnitude of interfacial roughness would impact the aforementioned interfacial fracture parameters, driving researchers to study its effect by investigating specimens with smooth interfaces[5] and artificial grooving interfaces[6]. In addition, it was found that unlike homogeneous materials, the bonded interface between dissimilar materials, owing to the

61 mismatching material properties, is always under the combination stress state of normal and
62 shear stresses even under the conditions of symmetric geometries and balanced external
63 loading. Through analyzing the interfacial fracture under various combinations of shear and
64 normal stresses[7,8], it was found that the fracture of the specimen along the interface always
65 corresponded to lower mixed mode stress intensity factor (SIF) ratios. On the contrary, the
66 cracks for higher mixed mode stress intensity factor ratios usually diverted to one side of the
67 interface[6,9]. In addition, the interfacial crack was inclined to kink into the weaker material
68 when there exists a large difference in material properties between two sides of the
69 interface[10]. In order to illustrate the fracture mechanism of an interface with respect to
70 different failure modes, i.e. the crack propagates along the interface and diverts to one side
71 of the interface, the nonlinear fracture theory, in which the strain localisation and nonlinear
72 characteristics of the interface are taken into account, should be introduced to replace LEFM
73 in the analysis of the interfacial fracture process.

74 For quasi-brittle materials, the fracture process zone (FPZ) lies in front of the crack tip,
75 and attracts significant concerns when studying the nonlinear response of an engineering
76 structure constructed with quasi-brittle materials during the fracture process. The effect of
77 the FPZ on the fracture parameters of concrete, as a type of quasi-brittle material, has been
78 extensively investigated in the last few decades. The size effect of the specific fracture
79 energy was found to be related with the FPZ properties[11,12], demonstrating that the FPZ
80 length in particular decreases rapidly when the crack propagates close to the top surface of
81 a specimen. Consequently, the local fracture energy was found to be not constant during the
82 whole fracture process, and instead decreased with the reduction of the FPZ length.

Combining the theoretical and experimental studies, a bilinear model on local fracture energy distribution was proposed to calculate the true specific fracture energy[13]. The significant effect of a varying FPZ on concrete fracture characteristics and the entire fracture process has engaged scientific and engineering communities. The relevant studies have been carried out through experimental investigations[14-16] and numerical simulations[17,18]. In addition, as one three-dimensional effect on fracture analysis, a coupled fracture mode was found to exist in cracked thick plate under shear or out-plane loading, and the intensity of the coupled mode was significantly influenced by the thickness of plate in three-dimensional finite element (FE) analysis[19-21]. However, the study on the evolution of the FPZ during the complete fracture process at a rock-concrete interface has been little reported. Regarding the rock-concrete interfacial fracture, it is worthwhile to point out that the derived fracture energy based on LEFM without considering the FPZ is less than that based on nonlinear fracture mechanics[22] by 83% . Therefore, it is significant to incorporate the study of the FPZ evolution at the rock-concrete interface when exploring the fracture mechanism and assessing the nonlinear response of a concrete structure constructed on bedrock.

Meanwhile, the crack propagation criteria in numerical methods have been widely investigated, which demonstrate the mechanism of crack growth in quasi-brittle materials like concrete. The criterion based on the maximum principle tensile stress has been adopted to simulate fracture processes of concrete by many researchers[22-24]. In this criterion, the accuracy of the calculated stress at a crack tip is largely influenced by element meshing and computational method. Therefore, to avoid getting misleading stress values at the crack tip, the stress intensity factor (SIF) - based criteria have been employed in the analyses of

105 fracture processes, and gained increasing attentions in the research communities. One type
106 of SIF-based criteria was proposed by Carpinteri and Massabó[25], in which the crack begins to
107 propagate when the stress singularity at the tip of the cohesive zone vanishes, i.e. SIF is
108 equal to 0. Later, this propagation criterion was also applied to study the crack growth in
109 concrete[26], other quasi-brittle materials[27] and composite materials with a rock-concrete
110 interface[6]. The other SIF-based criteria proposed by Dong[28] for determining the crack
111 propagation in concrete states that a crack initiates when the difference in the SIFs at the
112 crack tip caused by external loading and the cohesive stress reaches the initial fracture
113 toughness of concrete. The comparison of the two SIF-based criteria in the analysis of crack
114 propagation in concrete has been made, and the applicability of the criteria has been
115 discussed for concrete at various strength levels[29]. The concept of the SIF-based criterion
116 by Dong[28] was also coupled with the maximum circumferential stress criterion to investigate
117 mixed mode I-II fracture in concrete[30]. In addition, a generalised maximum tangential stress
118 criterion (GMTS) considering tensile stress was also employed to analyse the fracture of
119 granite under four-point bend loading[31]. However all studies mentioned above do not pertain
120 to specimens with blunt notches or U-notches. Considering the complex stress distribution at
121 the notch tip under mixed mode loading, a strain energy density (SED) - based[32] and crack
122 zone model (CZM) - based[33] fracture criterion were used to predict the critical load for blunt
123 U- and V-notched brittle specimens[34-36]. Meanwhile, together with the crack propagation or
124 fracture criteria, numerical methods such as the finite element method[28,37], extended finite
125 element method[26] and scaled boundary finite element method[6] have been employed to
126 simulate crack growth. These numerical methods provide an additional, effective tool to

study the FPZ evolution in both quasi-brittle materials and interfacial zones. Particularly, it is more convenient to employ numerical methods to study the size effect of the FPZ evolution which cannot be easily achieved through experimental investigations.

Therefore, the main objective of this paper is to employ a stress intensity factor (SIF) - based crack propagation criterion to investigate the FPZ evolution during the complete fracture process along a rock-concrete interface. Firstly, rock-concrete composite beams with different initial crack ratios are tested under TPB. Moreover, a DIC technique is used to investigate the FPZ evolution at the rock-concrete interface. The load versus crack mouth opening displacement (P-CMOD) curves (measured using one clip gauge mounted on the bottom surface of the specimen), crack surface opening displacement (measured through the digital image correlation technique) and the FPZ evolution based on the experimental results are compared with those from the numerical approach to verify the numerical method established in this study. Finally, the FPZ properties of composite beams with a rock-concrete interface are discussed based on the experimental and numerical results. It is expected that the experimental and numerical results presented here can lead to a better understanding of the FPZ evolution of a rock-concrete interfaces so that nonlinear fracture mechanics can be more efficiently employed to the analysis on crack propagation.

NUMERICAL METHOD

The crack propagation criterion which implements the initial fracture toughness has been adopted to successfully simulate the crack growth of mode I and mixed mode fracture in concrete[17,28,30]. This criterion for mode I crack propagation can be described by the following formula:

$$K_I^P + K_I^\sigma = K_{IC}^{ini} \tag{1}$$

where K_I^P and K_I^σ are the mode I SIFs caused by external loading and cohesive stress, respectively, and K_{IC}^{ini} is the initial fracture toughness of concrete. In order to calculate K_I^σ, the softening traction-separation law, i.e. the tension-softening relationship, should be adopted to determine the distribution of cohesive stress on a crack surface. Meanwhile, K_{IC}^{ini} can be regarded as a material property, which represents the capability of concrete to resist crack-initiation[38,39]. Based on the criterion, the whole fracture process of a beam under particular TPB can be described as a collection of critical crack-initiation states for a series of geometrically similar beams but with different effective crack lengths[28].

In the case of a rock-concrete interface, however, the crack propagation criterion mentioned above cannot be directly applied to investigate the crack growth at a rock-concrete interface. Calculating the SIFs at the crack tip for a bi-material interface is different from that for a homogeneous material. To make a distinction, the SIFs for a bi-material interface are always denoted as K_1 and K_2 instead of K_I and K_{II} in this paper. Generally, they are calculated through numerical methods, e.g. stress or displacement extrapolations. In this paper, the SIFs for a rock-concrete interface crack are calculated by the displacement extrapolation method using the ANSYS finite element code with the formulas shown as follows[40]:

$$K_1 = C \lim_{r\to 0} \sqrt{\frac{2\pi}{r}} \left[\delta_y(\cos Q + 2\varepsilon \sin Q) + \delta_x(\sin Q - 2\varepsilon \cos Q\right] \tag{2}$$

$$K_2 = C \lim_{r\to 0} \sqrt{\frac{2\pi}{r}} \left[\delta_y(\cos Q + 2\varepsilon \sin Q) - \delta_x(\sin Q - 2\varepsilon \cos Q\right] \tag{3}$$

where

$$C = \frac{2\cosh(\varepsilon\pi)}{(\kappa_1+1)/\mu_1 + (\kappa_2+1)/\mu_2} \tag{4}$$

$$Q = \varepsilon \ln r \tag{5}$$

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{\frac{\kappa_1}{\mu_1}+\frac{1}{\mu_2}}{\frac{\kappa_2}{\mu_2}+\frac{1}{\mu_1}}\right) \tag{6}$$

$$\mu_i = \frac{E_i}{2(1+v_i)} \qquad (i=1,2) \tag{7}$$

$$\kappa_i = \begin{cases} (3-v_i)/(1+v_i) & (Plain\ stress) \\ (3-4v_i) & (Plain\ strain) \end{cases} \tag{8}$$

E_i, v_i, δ_x and δ_y are the Young's modulus, Poisson's ratio, and the relative crack surface displacements in x and y directions, respectively.

As previously mentioned, crack growth along a rock-concrete interface is in fact a mixed mode fracture process. However, it should be noted that K_2 is far less than K_1 for a TPB beam[4]. As a consequence, the crack propagation along the rock-concrete interface in a TPB beam can be approximately considered as a mode I-dominated fracture process. Moreover, a bilinear relationship between cohesive stress (σ) and crack opening displacement (w) for a rock-concrete interface[41] is adopted in the numerical method, which is shown in Fig. 1. The crack opening displacement w_s and the corresponding cohesive stress σ_s at the breaking-point of the bilinear σ-w relationship is equal to $0.8G_f/f_t$ and $0.2f_t$, respectively. The stress-free crack opening displacement w_0 equals to $6G_f/f_t$. Here, f_t and G_f are the uniaxial tensile strength and fracture energy of a rock-concrete interface, respectively. A direct tension test was conducted to measure the uniaxial tensile strength of the rock-concrete interface. 100 mm × 100 mm × 200 mm prisms were prepared, which consisted of two

189 geometrically identical blocks, i.e. rock and concrete block, respectively. The uniaxial tensile
190 strength, f_t, is calculated from the following equation:

191 $$f_t = P_{max}/A \quad (9)$$

192 where P_{max} is the peak load, and A is the interfacial area.

193 In this paper, the crack propagation criterion is modified and extended to determine the
194 crack growth along the rock-concrete interface of a composite beam under TPB. The
195 modified crack propagation criterion for a rock-concrete interface can be expressed as
196 following:

197 $$K_1^P + K_1^\sigma = K_{1RC}^{ini} \quad (10)$$

198 In Eq. (10), K_1^P and K_1^σ can be calculated by inserting the relative crack surface
199 displacements δ_x and δ_y to Eqs. (2) and (3). Thus, the initial fracture toughness of the
200 rock-concrete interface, K_{1RC}^{ini} can be calculated by deriving δ_x and δ_y with respect to the
201 initial cracking load P_{ini}. Using the crack propagation criterion, the corresponding load at any
202 certain crack length can be derived. Thus, the complete crack propagation process can be
203 simulated, in which the details of the iterative process can be referred to elsewhere[28].

204 **EXPERIMENTAL PROGRAM**

205 To verify the proposed crack propagation criterion, a series of composite beams with a
206 rock-concrete interface were tested under TPB to investigate the fracture process. The
207 P-CMOD curves were derived from the experimental studies. In addition, taking a composite
208 beam as an example, the FPZ evolution during the complete fracture process was also
209 studied using a DIC technique.

210 **Experimental specimens**

The geometry of the rock-concrete composite beams and the test setup are illustrated in Fig 2(a). The dimensions of the composite beams are 500 mm × 100 mm × 100 mm. Each composite beam consists of two halves, a half concrete block and a half limestone block. It should be noted that to make a composite beam, concrete was cast against the half rock block with a natural surface which was obtained after a TPB test on a pre-notched rock beam (See Fig 2(b)). The rock beams have the same dimensions as the composite ones, but with different initial crack lengths. Five series of composite beams with a_0/D from 0.2 to 0.6, denoted as TPB20 to TPB60, were prepared, and three parallel composite beams were produced for each series. Grade 42.5R Portland cement, crushed stone with the maximum size of 10 mm and medium-size river sand were used for making concrete with a mix of 1:0.62:1.81:4.20 by weight (cement : water : sand : aggregate). The composite specimens were demolded one day after casting and then cured in an environment of 23℃ and 90% RH for 28 days. The material properties of rock, concrete and the interface were measured with the results presented in Table 1, in which E, v, f_c and f_t denote the elastic modulus, Poisson's ratio, compressive strength and splitting tensile strength, respectively.

Three-point bending tests

The three-point bending tests on the rock-concrete composite beams were performed using a 250kN closed-loop servo-hydraulic testing machine (MTS) under a displacement control loading mode at a rate of 0.012 mm/min. Both the displacement at the loading point and the CMOD were measured using clip gauges. In order to measure P_{ini}, four strain gauges were symmetrically attached at the pre-notch tip on both sides of a composite beam. Strain gauges and clip gauges, together with a load cell were connected to an Integrated

233 Measurement & Control (IMC) dynamic date acquisition device. The setup for the TPB
234 testing is illustrated in Fig 3.

235 The fracture parameters of a composite beam with a rock-concrete interface including K_{1RC}^{ini},
236 P-CMOD curve and G_f were derived from the results of the three-point bending tests. Based
237 on LEFM, in order to calculate the initial fracture toughness, P_{ini} should be determined.
238 When a crack initiates and starts to propagate, the measured strain on the two sides of the
239 crack will suddenly and significantly decrease due to the release of fracture energy.
240 Therefore, P_{ini} can be determined according to the variation of the measured strain at the tip
241 of a pre-crack. Meanwhile, G_f of the rock-concrete interface was derived from the
242 load-deformation curves, which is the same procedure to concrete suggested by RILEM[42].
243 The deformation at the loading point can be measured using the clip gauge mounted on the
244 top of the specimen (See Fig. 3). G_f can be calculated from the following equation:

245 $$G_f = \frac{W_0 + 2P_w\delta_0}{A_{lig}} \tag{11}$$

246 where W_0 is the area of load-deformation curves, P_w is the self-weight of the composite
247 specimen, δ_0 is the deformation when the load decreases to 0, and A_{lig} is the ligament area
248 of the composite specimen.

249 **Digital image correlation (DIC) test**

250 In order to verify the proposed crack propagation criterion and investigate the FPZ
251 evolution along a rock-concrete interface during the entire fracture process, the
252 displacement on the cracking surface of beam TPB30-1 was studied using a DIC technique,
253 with the test setup shown in Fig. 4. Before performing the TPB test, white and black spray
254 paint was used to create a speckle pattern on one side of the potential cracking surface of a

255 TPB30-1 beam. A digital camera with a resolution of 1024×768 pixels and a host-computer
256 were used in the DIC test to measure the displacement on the crack surface. Images of the
257 deformation were snapped by the digital camera every half a second and stored in a
258 host-computer. The deformation field near the crack tip can be composed by comparing
259 images of the crack surface before and during the loading. Therefore, the complete crack
260 propagation process can be tracked using the opening and sliding displacements at the
261 rock-concrete interface obtained directly by using the DIC imaging.

262 The computational domain is selected based on the potential crack propagation route,
263 in which one analysis point was picked from every five pixels in both u and v directions (see
264 Fig. 5). For beam TPB30-1, a total of 26128 (142×184) points were picked in the
265 computational domain. The analysis lines M_iN_i (i = 1, 2, …, 184) were set for every five
266 pixels apart in the v direction, in which the u and v displacements of each analysis point can
267 be determined. In the numerical procedure, the u and v displacements of the analysis points
268 located on M_1N_1 to M_iN_i lines were determined at each load level. According to the
269 tension-softening constitutive law of a rock-concrete interface proposed by Dong[43], the
270 critical crack opening displacement w_0 for beam TPB30-1 equals 0.0844 mm. Thus, the FPZ
271 length and opening/sliding displacements can be determined at each load level. In addition,
272 the trajectory of crack growth can be observed through the strain-contour diagram in the
273 computational domain.

274 **EXPERIMENTAL RESULTS AND NUMERICAL VERIFICATIONS**

275 **P-CMOD curves**

276 It was observed from the experiment that the fracture of all composite beams was

caused by crack propagation along their rock-concrete interface under TPB. The experimental results, which are listed in Table 2, show that initial fracture toughness of mode II, K_{2RC}^{ini}, is much less than that of mode I, K_{1RC}^{ini}. In order to verify the proposed propagation criterion, comparisons of the experimental and numerical P-CMOD curves are made, and the results are shown in Fig. 6. Moreover, relative material properties used in numerical simulation are listed in Table 3. It can be seen that the numerical results are in a reasonably good agreement with the experimental ones, indicating the validity of the proposed crack propagation criterion in the analysis of interfacial crack propagation.

FPZ evolution

In this paper, the crack propagation process of beam TPB30-1 was studied using the DIC technique. By considering the two loading stages, i.e., $P_1 = P_{max}$ and $P_2 = 11\%P_{max}$ at the post-peak branch, i.e. the strain-softening branch, as examples, the FPZ evolution and corresponding opening/sliding displacements are illustrated in Fig. 7. Moreover, the strain fields at the two loading stages are illustrated in Fig. 8. It can be seen from Fig. 7 that the crack sliding displacement is very small compared with the crack opening displacement, which indicates that the rock-concrete interface fracture under TPB is predominantly dominated by mode I fracture.

The FPZ evolution of beam TPB30-1 is also simulated using the numerical method based on the proposed crack propagation criterion. A comparison of the crack opening displacement field is made with respect to the loading levels P_1 and P_2 between the DIC and numerical results, which are shown in Figs. 9(a) and (b). Moreover, the FPZ evolution in the complete fracture process is also analyzed, and comparisons between numerical and

299 experimental results are then made (see Fig. 10). It can be seen from Figs. 9 and 10 that the
300 numerical results are in a reasonably good agreement with the experimental ones, which
301 verifies the proposed numerical method in the analysis of FPZ evolution at a rock-concrete
302 interface.

303 **DISCUSSIONS ON FPZ EVOLUTION**

304 The experimental results obtained under the current laboratory conditions in this study
305 may to certain degree affect the expansion of the conclusions to practical engineering
306 structures due to the size effect. From the viewpoint of structure reliability assessment, it is
307 expected to have a better understanding of the size effect of FPZ evolution in quasi-brittle
308 materials and along interfaces, so that the fracture mechanism of practical structures can be
309 revealed reasonably and the failure process can be predicted precisely. In this section, the
310 verified numerical method is used to investigate the FPZ evolution at a rock-concrete
311 interface. Also the material properties used in the following numerical analyses are listed in
312 Table 3.

313 **Size effect on FPZ evolution**

314 In order to investigate the effect of a_0/D on the FPZ evolution, the fracture process of the
315 T series rock-concrete composite beams with the same size but different notch/depth (a_0/D)
316 ratios is studied using numerical analysis. The geometries of the T series composite beams
317 and the numerical results, including the initial cracking load P_{ini}, peak load P_{max} and full FPZ
318 length L_{FPZ}^{max}, are listed in Table 4. Fig. 11 shows the FPZ evolutions for the five T-series
319 composite beams. It can be seen that the FPZ length, l_{FPZ}, linearly increases until the full
320 FPZ length L_{FPZ}^{max} is reached, and then decreases rapidly for all beams. With the increase of

321 a_0/D, i.e. the decrease of ligament length, the maximum FPZ length that can be attained
322 decreases but the corresponding ratio of $\triangle a/(D - a_0)$ increases.

323 In addition, the effect of specimen size on the FPZ evolution is also studied on the L
324 series composite beams with the same a_0/D but varied sizes (see Table 5). It should be
325 noted that the whole fracture process of beams with a high depth cannot be obtained due to
326 large self-weights of the specimens. Therefore the self-weight of the L series composite
327 beams are disregarded in the numerical analyses. The FPZ evolutions of the L series beams
328 are illustrated in Fig. 12. It can be seen that the ratio of L_{FPZ}^{max} to the ligament length ($D - a_0$)
329 also gradually decreases as the depth D increases from 100 mm to 20000 mm (20 m). The
330 FPZ length begins to decrease after the FPZ fully develops in cases of small specimens, e.g.
331 Specimens L100 and L150. However, the declining tendency becomes slow with the
332 increase of specimen size. Particularly, in the cases of D = 10 m and 20 m, the variations of
333 the FPZ length are approximately steady as crack propagates after reaching the full FPZ.
334 The results indicate that the FPZ evolution at a rock-concrete interface is largely dependent
335 on the specimen size. Therefore, for concrete structures constructed on rock with a large
336 ligament length, the FPZ will move forward with an almost constant length, i.e. the full FPZ
337 length, under mode I-dominated fracture.

338 In order to explore the mechanism of FPZ evolution, the FPZ evolutions in the T and L
339 series specimens are illustrated together in Fig. 13, which are arranged according to the
340 ligament lengths. It can be observed that the ratio of L_{FPZ}^{max} to ($D - a_0$) decreases as the
341 ligament length ($D - a_0$) increases. To further verify the specimen size effect on the FPZ
342 evolution at a rock-concrete interface, the S series composite beams, which have the same

343 ligament length but different a_0/D ratios, are also analysed in the study. The geometries of
344 the S series beams and numerical results are presented in Table 6. The FPZ evolutions of
345 the S series beams are illustrated in Fig. 14. It can be seen that the FPZ length remains
346 equal for all the S series beams even though they have different a_0/D ratios. Therefore,
347 based on the analyses of FPZ evolution in the L, T and S series beams, it can be concluded
348 that the ligament length indeed affects the interfacial FPZ evolution and results in the
349 specimen size effect of FPZ (??). Meanwhile, it is also found that the ratio of L_{FPZ}^{max} to $(D - a_0)$
350 decreases with the increase of the ligament length $(D - a_0)$, which indicates that LEFM is
351 appropriate for fracture analysis of structures with larger ligament length for the purpose of
352 simplified calculation.

353 **Effect of specimen size on K_2/K_1 ratio**

354 Under TPB, due to the materials being different on both sides of a crack, the
355 rock-concrete interface is under mixed mode fracture rather than under single mode fracture.
356 Therefore, the ratio of K_2/K_1 would vary as the crack propagates along the interface. For the
357 T series specimens, K_2/K_1 versus a/D relationships are illustrated in Fig. 15. It can be seen
358 that these curves almost coincide with each other although they have different starting
359 points, i.e. different a_0/D ratios. Thus, in the case of specimens with the same size, the K_2/K_1
360 versus a/D curve with respect to the smallest a_0/D can be regarded as the envelope of those
361 curves for the specimens with larger a_0/D ratios. The maximum value of K_2/K_1 is 0.172,
362 which corresponds to a/D = 0.62.

363 In addition to the effect of a_0/D, the effect of specimen size on the K_2/K_1 ratio is also
364 studied. Fig. 16(a) illustrates the variations of K_2/K_1 for Specimens L100, L150, L200 and

L250, and indicates that the value of K_2/K_1 increases with the increase of specimen size. However, it should be noted that, comparing with the initial K_2/K_1 corresponding to the initial crack length a_0, the increase is not significant for the maximum value of K_2/K_1. Taking Specimen L100 as an example, the initial K_2/K_1 is 0.15 and the maximum is 0.175. Therefore, the initial K_2/K_1 without crack propagation can approximately reflect the maximum ratio for mixed mode during the fracture process although the mode varies as the crack propagates. Moreover, the value of K_2/K_1 decreases rapidly when the crack is close to the top surface of a rock-concrete composite beam, leading the rock-concrete interface fracture to be mode I dominated at that moment.

On the other hand, the effect of a_0/D on the K_2/K_1 ratio also attracts attention of researchers who would measure the fracture parameters at rock-concrete interface under mode I fracture. Upon this point, the LEFM analysis is conducted to investigate the effect of a_0/D on the initial K_2/K_1 ratio in the initial cracking state since no crack propagation happens yet. In the analysis, the external load is set as a constant value, and the initial crack length is increased step by step. At each step, the corresponding K_2/K_1 at the crack tip is calculated. Taking Specimen T20 as an example, the variations of K_2/K_1 during the complete fracture process without considering the action of the cohesive stress are shown as the curve for T20L in Fig. 15. In fact, the curve presents the variations of the initial K_2/K_1 for the rock-concrete composite beams with the same size as T20 but varying a_0/D from 0.2 to 1.0. It can be seen from the curve that the value of K_2/K_1 increases slowly from a_0/D = 0.2 until the peak is reached, and then reduces completing the whole fracture process (??). The ratio of K_2/K_1 reaches the peak value of 0.12 at point A when a/D is about 0.56. Fig. 16(b)

387 illustrates the variations of K_2/K_1 with respect to Specimens L100, L150, L200 and L250,
388 without considering the effect of the cohesive force. Similar to the curve for T20L in Fig. 15,
389 these curves also represent the variations of the initial K_2/K_1 for the specimens with the
390 same size as L100, L150, L200 and L250 but varying a_0/D from 0.2 to 1.0. Based on the
391 results in Fig. 16(b), it can be concluded that the initial K_2/K_1 is largely affected by the
392 specimen size. With the increase of specimen size, the initial K_2/K_1 increases for the same
393 a_0/D. Meanwhile, the value of K_2/K_1 decreases rapidly when the crack is close to the top
394 surface of the specimen, which shows a similar tendency to the variations of K_2/K_1
395 considering the effect of FPZ. Although the pure mode I fracture can be approximately
396 obtained for a TPB composite beam when a/D is close to 1, it is not convenient to conduct
397 experiment in the lab because the large pre-crack length makes a specimen break easily.
398 Therefore, it is more reasonable to test the specimens with small a_0/D and small geometry
399 size to obtain the mode I fracture at a rock-concrete interface.

400 The peak values of K_2/K_1 for different specimens with/without considering the effect of
401 FPZ are presented in Table 7. It can be seen from the table that the maximum values of
402 K_2/K_1 of all specimens are reached when a/D is around 0.57. The difference between
403 maximum K_2/K_1 value calculated with and without considering the effect of FPZ increases as
404 the beam depth increases. This can be explained by the fact that higher beams have lager
405 FPZ length so that the restriction of the cohesive stress on K_1 is more significant.

406 In order to further verify the effect of specimen size on the initial K_2/K_1, the analyses,
407 based on LEFM, are conducted on various sizes of TPB specimens with a_0/D = 0.3. Fig. 17
408 illustrates the initial K_2/K_1 versus specimen depth D relationship. It can be seen from this

409 figure that the value of K_2/K_1 increases as the specimen depth increases, and the
410 relationship can be approximately expressed by using a logarithmic function. Curve fitting is
411 hence applied and the corresponding logarithmic function can be obtained as follows:

412 $$K_2/K_1 = 0.1 \log(D) - 0.09 \quad (12)$$

413 According to the results shown in Fig. 17, in the case of rock-concrete interfacial fracture,
414 the value of K_2/K_1 is close to 0.3 for large size specimens under TPB. Combining the results
415 in Fig. 16 indicates that the ratio of K_2/K_1 would further increase during crack propagation
416 due to the effect of FPZ. As a consequence, in the case of a concrete dam with a crack
417 along the interface between the concrete dam and the rock foundation, the crack may divert
418 into the rock foundation after propagating certain length due to the increase of K_2/K_1 during
419 the fracture process. (Why not propagate into concrete dam if concrete is weaker?)

420 **CONCLUSIONS**

421 A crack propagation criterion for a rock-concrete interface is proposed to investigate the FPZ
422 evolution in rock-concrete composite beams under TPB. The experiments including the TPB
423 testing and the DIC technique are conducted to measure the P-CMOD curves and the FPZ
424 evolution. These results then are compared with the numerical simulations to verify the
425 proposed criterion. Moreover, the numerical method combining with the proposed criterion is
426 employed to study the effect of the ligament length on the FPZ evolution and the variations
427 of fracture model during crack propagation for the rock-concrete interface fracture. The
428 following conclusions can be drawn:

429 (a) By comparing with the P-CMOD curves and the FPZ evolutions from the experimental
430 investigations, the numerical results show a reasonably good agreement, which verifies

the proposed criterion in the analysis of rock-concrete interfacial fracture. Therefore, if the elastic moduli E of concrete and rock, the interfacial uniaxial tensile strength f_t, the interfacial fracture energy G_f and the initial fracture toughness K_{1RC}^{ini} are given, the complete fracture process at rock-concrete interface under TPB can be predicted based on the proposed numerical model.

(b) The ligament length significantly affects the FPZ evolution at a rock-concrete interface under TPB. Both experimental and numerical results show that the FPZ length linearly increases as crack propagates until the maximum FPZ length is reached, and decreases thereafter. In the case of a large ligament length, with the maximum of 14 m in this study, the decreasing tendency of the FPZ is slow and keeps an approximately plateau after its full development.

(c) The ratio of K_2/K_1 varies during the interfacial crack propagation under TPB, slowly increasing first and then decreasing. For specimens with different sizes, the maximum of K_2/K_1 can be achieved when a/D is approximate 0.57. When the crack is close the top surface of the specimen, the ratio of K_2/K_1 sharply decreases to 0, which indicates the fracture of the composite specimen is dominated by mode I failure. Specimen size affects the variations of K_2/K_1 during interfacial crack propagation. Larger K_2/K_1 ratios can be obtained for higher specimens under the same a/D. The initial K_2/K_1 ratio versus specimen depth relationship can be expressed using a logarithmic function.

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Appendix I Tables

Table 1: Mechanical properties of rock, concrete and their interface

Material	ρ (kg/m^3)	E (GPa)	ν	f_c (MPa)	f_t (MPa)
Concrete	2400	33.29	0.204	38.96	—
Rock	2668	64.39	0.198	119.2	—
Interface	—	—	—	—	1.371

Table 2: Experimental results of TPB series beams

Specimen	$S\times D\times B$ (mm×mm×mm)	a_0 (mm)	P_{ini} (kN)	P_{max} (kN)	K_{1RC}^{ini} (MPa·m$^{1/2}$)	K_{2RC}^{ini} (MPa·m$^{1/2}$)	G_f (N/m)
TPB20-1	400×100×100	20	2.322	2.644	0.332	-0.020	28.256
TPB20-2		22	2.395	2.657	0.373	-0.025	23.373
TPB20-3		22	3.467	3.725	0.539	-0.037	40.342
TPB30-1		30	1.730	2.200	0.335	-0.036	19.300
TPB30-2		31	2.094	2.385	0.412	-0.034	—
TPB30-3		30	1.326	2.068	0.254	-0.021	19.516
TPB40-1		42	1.487	1.746	0.392	-0.036	19.339
TPB40-2		40	1.741	2.162	0.434	-0.039	—
TPB50-1		50	1.154	1.409	0.385	-0.036	25.781
TPB50-2		50	0.755	1.087	0.252	-0.024	21.574
TPB50-3		51	1.075	1.493	0.371	-0.035	31.062
TPB60-1		60	0.577	0.748	0.296	-0.027	11.128

Table 3: Fracture parameters used in the numerical simulations

Specimen	K_{1RC}^{ini} (MPa·m$^{1/2}$)	f_t (MPa)	E (GPa)		ν		G_f (N/m)
			Concrete	Rock	Concrete	Rock	
TPB20-1	0.332	1.371	33.29	64.39	0.204	0.198	28.256
TPB30-1	0.335						19.300
TPB40-1	0.392						19.339
TPB50-1	0.385						25.781
TPB60-1	0.296						11.128
T-, L-, S-series	0.335						19.300

578 Table 4: Dimensions and numerical results of T series composite beams

Test Series	$S\times D\times B$ (mm×mm×mm)	a_0/D	P_{ini} (kN)	P_{max} (kN)	L_{FPZ}^{max} (mm)
T20	400×100×100	0.2	2.140	2.795	66.0
T30		0.3	1.640	2.160	58.0
T40		0.4	1.250	1.630	52.0
T50		0.5	0.930	1.185	44.0
T60		0.6	0.650	0.795	36.0

579

580 Table 5: Dimensions and numerical results of L series composite beams

Specimen	$S\times D\times B$ (mm×mm×mm)	a_0 (mm)	a_0/D	P_{ini} (kN)	P_{max} (kN)	L_{FPZ}^{max} (mm)
L100	400×100×100	30	0.3	1.640	2.175	58.0
L150	600×150×100	45		1.980	2.810	84.2
L200	800×200×100	60		2.230	3.325	110.0
L250	1000×250×100	75		2.430	3.755	132.0
L500	2000×500×100	150		3.380	6.270	232.0
L1000	4000×1000×100	300		5.410	9.575	392.0
L2000	8000×2000×100	600		7.850	14.320	610.0
L5000	20000×5000×100	1500		11.510	24.640	967.5
L10000	40000×10000×100	3000		16.300	36.350	1260.0
L20000	80000×20000×100	6000		20.350	52.750	1520.0

581

582 Table 6: Dimensions and numerical results of S series composite beams

Specimen	$S\times D\times B$ (mm×mm×mm)	a_0/D	$D-a_0$ (mm)	P_{ini} (kN)	P_{max} (kN)	L_{FPZ}^{max} (mm)
S1	500×125×100	0.2	100	2.400	3.240	81.2
S2	576×144×100	0.3		1.950	2.730	82.0
S3	672×168×100	0.4		1.550	2.245	82.0
S4	800×200×100	0.5		1.200	1.755	81.2
S5	1000×250×100	0.6		0.800	1.235	81.0

583

584 Table 7: Maximum values of K_2/K_1 of L series composite (??) beams

Specimen	Without cohesive force		With cohesive force		Difference of K_2/K_1 with/without cohesive force
	a/D	K_2/K_1	a/D	K_2/K_1	
L100	0.58	0.12032	0.58	0.17526	0.05494
L150	0.55	0.13783	0.55	0.20218	0.06435
L200	0.58	0.15034	0.61	0.22291	0.07257
L250	0.56	0.16026	0.56	0.23929	0.07903

Appendix II Figures

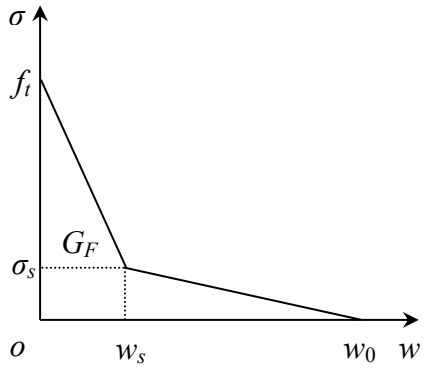
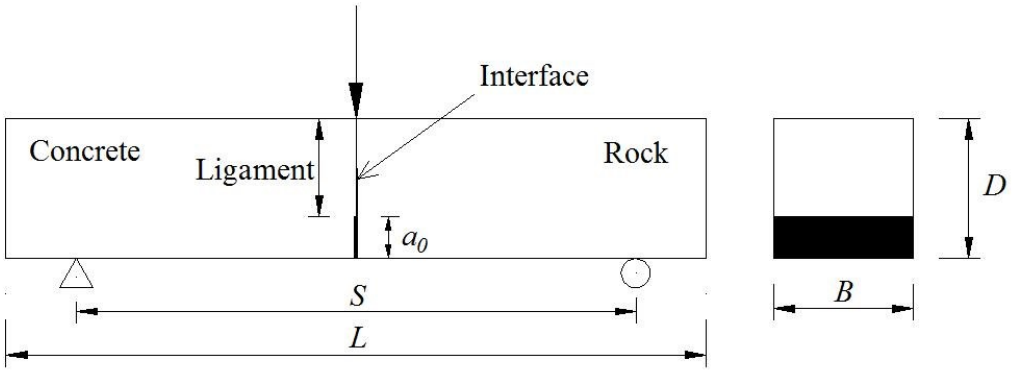


Fig.1. Bilinear stress - crack width curve for softening of concrete



(a) Specimen geometry

(b) Specimen preparation

Fig. 2. Geometry and preparation of rock-concrete composite specimens

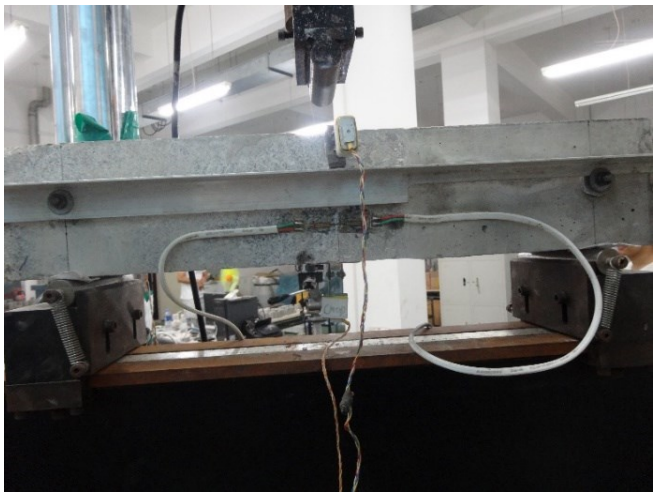


Fig. 3. Three-point bending test setup

Fig. 4. DIC test setup

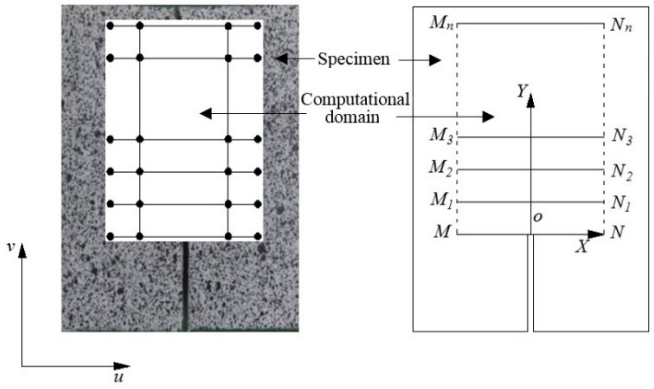
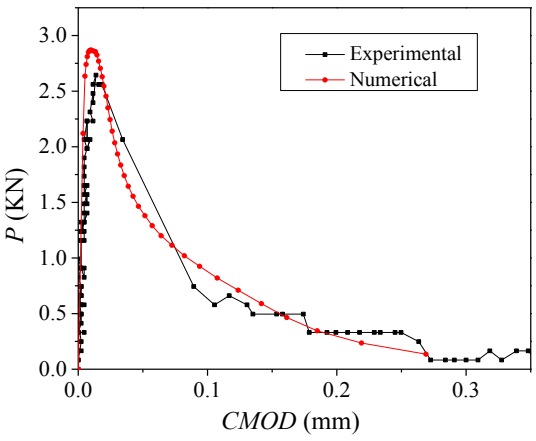
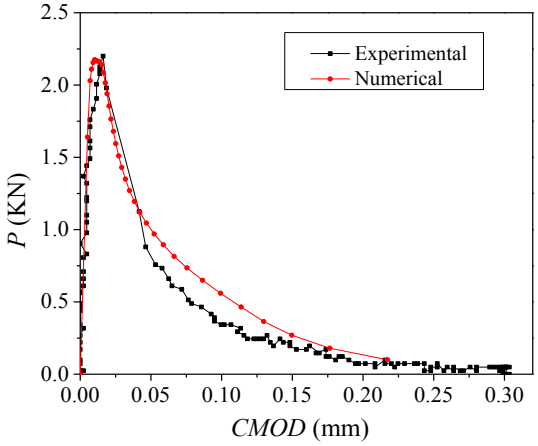


Fig. 5. Computational grids in the DIC test

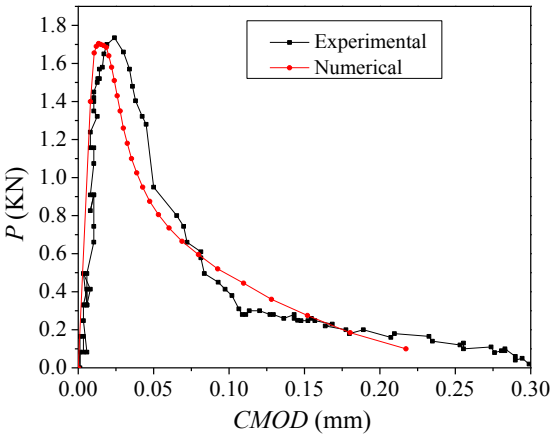


602

603 (a) TPB20-1



(b) TPB30-1



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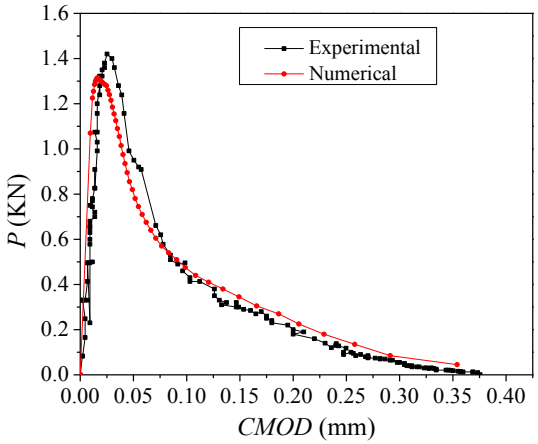
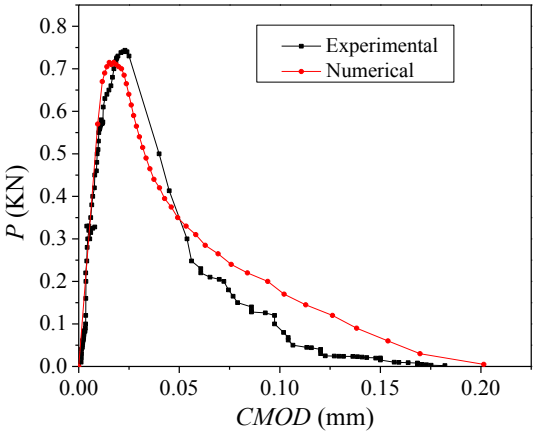
605 (c) TPB40-1



(d) TPB50-1



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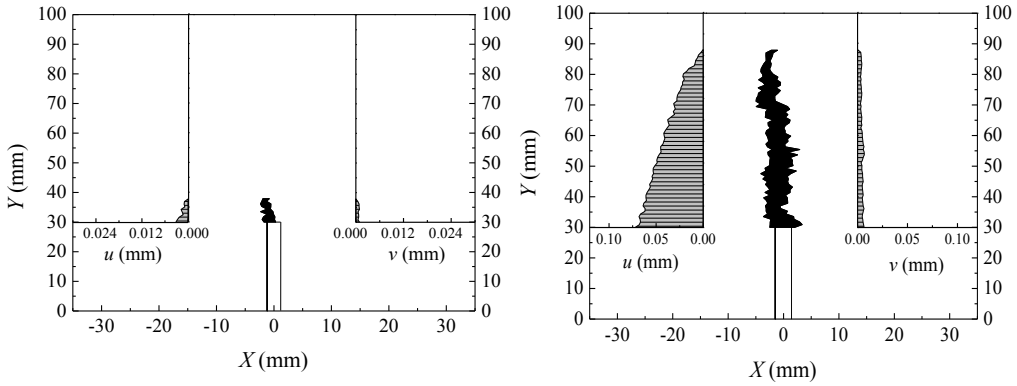
607 (e) TPB60-1

608 Fig. 6. Comparison of experimental and numerical P-CMOD curves (vertical axis is kN)



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610 (a) Loading level P_1 (b) Loading level P_2

611 Fig. 7. FPZ evolution at various loading stages

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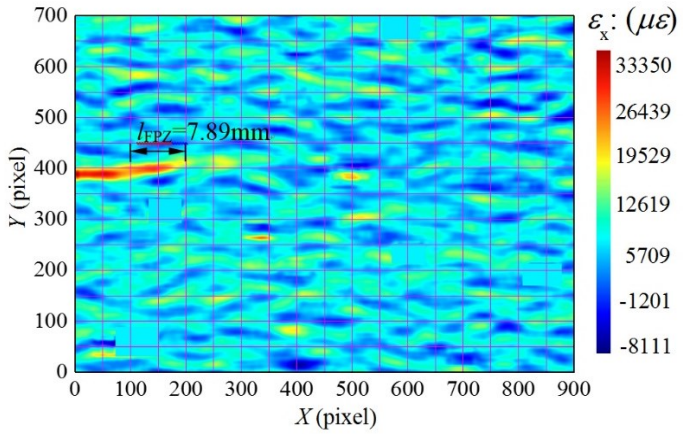
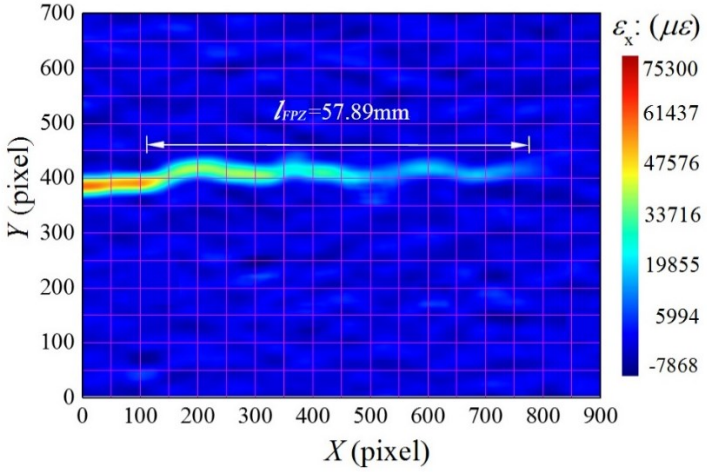


613

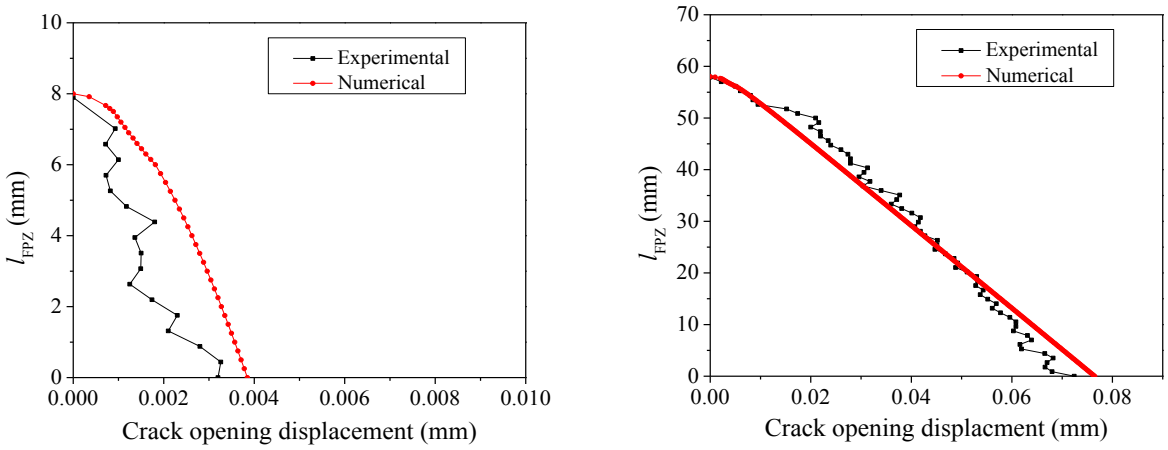
614 (a) Loading level P_1



615

616 (b) Loading level P_2

617 Fig. 8. Strain contours at various loading stages



(a) Loading level P_1 (b) Loading level P_2

Fig. 9. Comparison of experimental and numerical crack opening displacement distributions at various loading stages

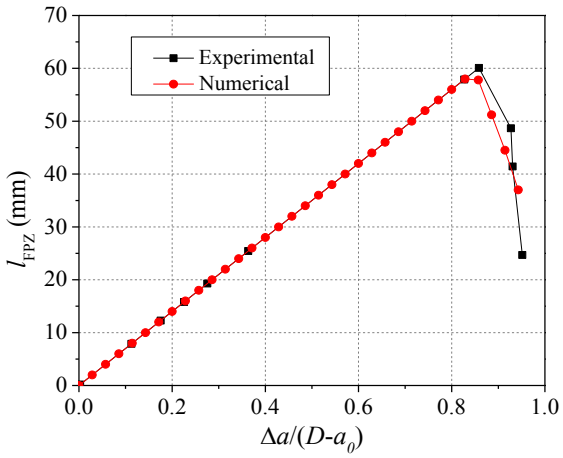
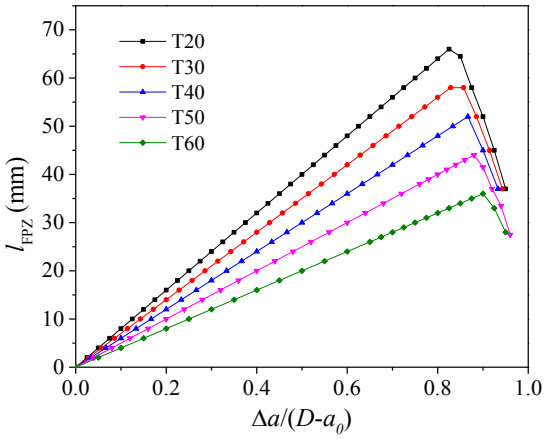
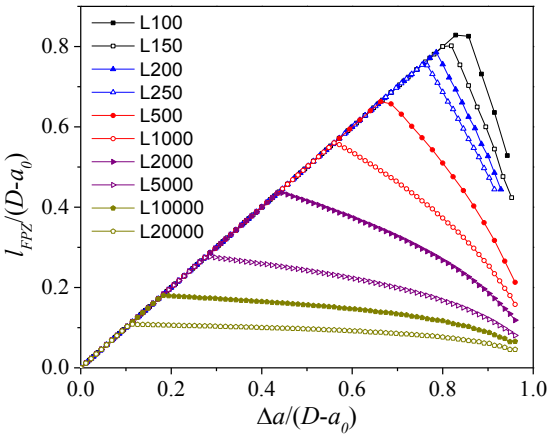


Fig. 10. Comparison of experimental and numerical FPZ evolutions for beam TPB30-1



628

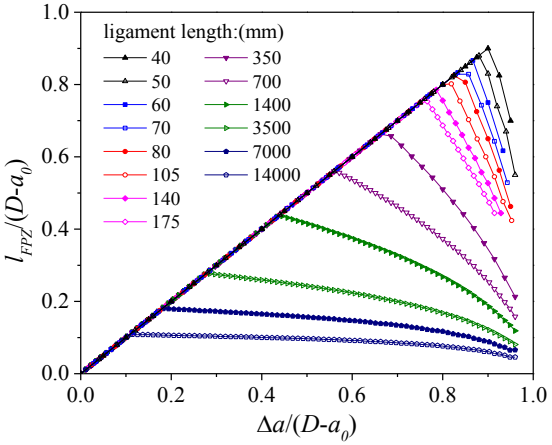
629 Fig. 11. Variations of FPZ length for T-series beams



630

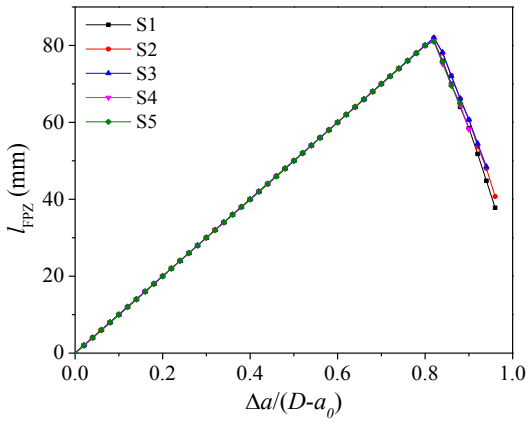
631 Fig. 12. Variations of FPZ length for L-series beams

632



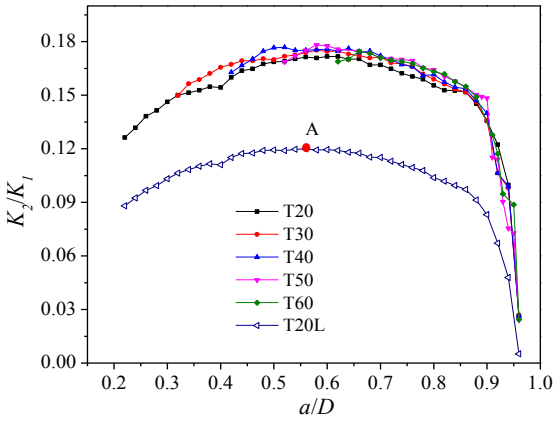
633

634 Fig. 13. FPZ evolutions for different ligament lengths



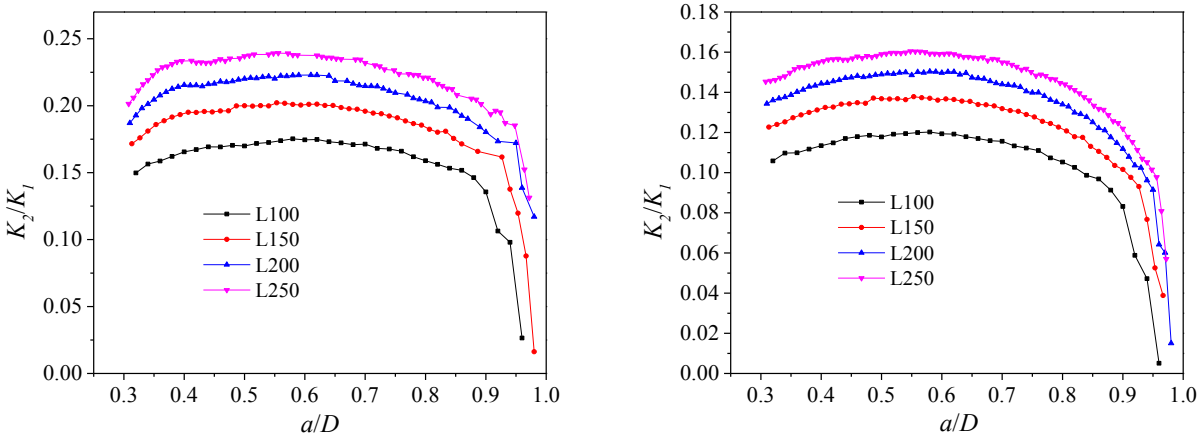
635

636 Fig. 14. Variations of fracture process zone for S-series beams



637

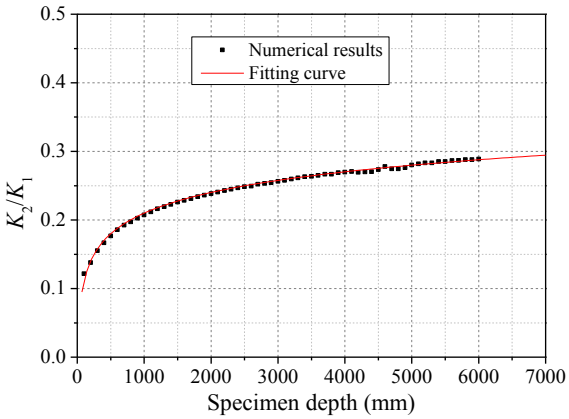
638 Fig. 15. K_2/K_1 variations during the complete fracture process for T-series beams and T20L



639

640 (a) With consideration of cohesive stress (b) Without consideration of cohesive stress

641 Fig. 16. K_2/K_1 variations during the complete fracture process for L-series beams



642

643 Fig. 17. K_2/K_1 versus beam depth with the fitting curve

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645